Machine Perception: CNN

1 1D Convolution

In this question we consider the 1-Dimensional case of convolution layer with zero padding. As illustrated in Fig. 1, the input feature vector $\boldsymbol{x} \in \mathbb{R}^5$ is convolved by a kernel $\boldsymbol{k} \in \mathbb{R}^2$ to $\boldsymbol{y} \in \mathbb{R}^5$.

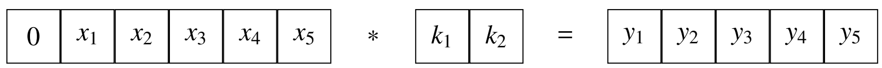

Figure 1: 1D convolution with zero padding.

(a) Write down the formula to compute each element of the output y_i.

(b) Convolution is a linear operator which can be written in the form of matrix multiplication $\boldsymbol{y} = \boldsymbol{M}\boldsymbol{x}$. Derive the convolution matrix $\boldsymbol{M}$ as a function of the kernel elements k_1, k_2.

(c) The graph in Figure.2 (a) represents a fully connected layer, which also performs matrix multiplication. Based on the convolution matrix $\boldsymbol{M}$, draw the connections for the convolution layer in Figure.2 (b) and label the weight for each edge. *Hint: discard zero-entries.*

(d) According to Figure.2 (b), name the differences between convolution layers and fully connected layers.

(a) fully connected layer

(b) convolution layer

Figure 2: Fully connected layer and convolution layer

2 Forward and backward

1. In the following is a 2D convolution layer followed with a ReLU activation layer and a max pooling layer. Calculate outputs of these layers $\mathbf{Y}$, $\mathbf{Y}'$ and $\mathbf{Y}''$ during the forward pass.

0	0	1	1	0
0	1	0	1	1
1	1	1	1	0
1	0	1	1	1
1	1	0	1	1

$\mathbf{X}$

1	0	1
0	1	0
0	1	0

$\mathbf{K}$

$\mathbf{Y} = \mathbf{X} * \mathbf{K}$

$\mathbf{Y}' = \mathbf{ReLU}(\mathbf{Y})$

$\mathbf{Y}'' = \mathbf{Pool}(\mathbf{Y}')$

2. Given the $\partial E/\partial \mathbf{Y}''$, calculate the partial deriviative of the error w.r.t $\mathbf{Y}'$,$\mathbf{Y}$,$\mathbf{K}$ and $\mathbf{X}$ for back-propagation.

$\partial E/\partial \mathbf{X}$

$\partial E/\partial \mathbf{K}$

$\partial E/\partial \mathbf{Y}$

$\partial E/\partial \mathbf{Y}'$

1

$\partial E/\partial \mathbf{Y}''$